Hello Students,

This resource packet includes a project that you can work on independently at home. You should also have project packets for some of the other courses you are enrolled in. These projects are standards-aligned and designed to meet the Remote Learning instructional minutes guidelines by grade band.

<table>
<thead>
<tr>
<th>High School Algebra Project: Maximizing Profit: Selling Boomerangs</th>
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<tbody>
<tr>
<td><strong>Estimated Time</strong></td>
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</table>
| **Grade Level Standard(s)** | ● A-CED - Create equations that describe numbers or relationships  
● A-REI - Reasoning with Equations and Inequalities |
| **Caregiver Support Option** | Listen to student explanations and help them talk through their interpretations. |
| **Materials Needed** | Pencil, straight edge (ruler, ID card, edge of a notebook), calculator (optional) |
| **Question to Explore** | How do businesses maximize profit? What information is necessary and how can it be used? |
| **Student Directions** | Read each situation carefully. Write a system of equations or inequalities that represents the given information. Solve the systems and interpret your solutions in terms of the context of the problem. Unless otherwise indicated, use a separate sheet of paper for your responses. |
Activity 1: Dimes and Quarters
A system of linear equations or inequalities are two or more equations/inequalities that contain the same variables. When writing a system of linear equations or inequalities it is important to clearly define the variables and use appropriate units of measurement. For the situation below, choose the system that can be used to represent the given information. Then, explain your choice, being sure to interpret the terms in the context of the problem.

The only coins that Alexis has are dimes and quarters.

- Her coins have a total value of $5.80.
- She has a total of 40 coins.

Which of the following systems of equations can be used to find the number of dimes, \( d \), and the number of quarters, \( q \), Alexis has? Explain your choice.

\[
\begin{align*}
\text{A.} & \quad \begin{cases} 
  d + q = 5.80 \\ 
  40d + 40q = 5.80 
\end{cases} & \quad \text{B.} & \quad \begin{cases} 
  d + q = 40 \\ 
  0.25d + 0.10q = 5.80 
\end{cases} \\
\text{C.} & \quad \begin{cases} 
  d + q = 5.80 \\ 
  0.10d + 0.25q = 40 
\end{cases} & \quad \text{D.} & \quad \begin{cases} 
  d + q = 40 \\ 
  0.10d + 0.25q = 5.80 
\end{cases}
\end{align*}
\]

Activity 2: How much folate?

Use the following information to answer the questions below: Sara’s doctor tells her she needs between 400 and 800 milligrams of folate per day, with part coming from her diet and part coming from a multivitamin. Each multivitamin contains 50 mg of folate, and because of the inclusion of other vitamins and minerals, she can only take a maximum of 8 tablets per day.

A. What are the possible combinations of \( n \), number of vitamins taken, and \( a \), the amount of dietary folate, which will give Sara exactly the minimum of 400 mg of folate each day? Complete the table to show your answers. (Hint for completing the table: think about what is the fewest number of pills she could take? What is the highest number?)

<table>
<thead>
<tr>
<th>( n ), number of tablets</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( 50n ), total folate (in mg) from tablets</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a ), total folate from diet (in mg)</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
B. What are the possible combinations of vitamin tablets and dietary folate which give the maximum of 800 mg of folate each day? Complete the table to show your answer.

<table>
<thead>
<tr>
<th>$n$, number of tablets</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$50n$, total folate (in mg) from tablets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a$, total folate from diet (in mg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C. Now use your tables from parts A and B to express your answers as a system of three inequalities. Explain the meaning behind each inequality. (Hint: see the section at the end with definitions and examples for inequalities.)

a. 

b. 

c. 

D. Create a graph on the coordinate plane provided of $a$ versus $n$. 
Activity 3: Boomerangs (Modified from Maximizing Profits: Selling Boomerangs, map.mathshell.org)

Phil and Cath make and sell boomerangs for a school event. The money they raise will go to charity. They plan to make them in two sizes: small and large. Phil will carve them from wood. The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve. Phil has a total of 24 hours available for carving. Cath will decorate them. She only has time to decorate 10 boomerangs of either size. The small boomerang will make $8 for charity. The large boomerang will make $10 for charity. They want to make as much money for charity as they can. How many small and large boomerangs should they make? How much money will they then make? (If you get stuck, see the hints at the end of the packet.)

A. Setting up the system: Using the information given, write a system of inequalities that represents the number of small boomerangs, \( x \), and the number of large boomerangs, \( y \), that can be made. In addition, include an equation representing the profit from selling the boomerangs.

B. Error Analysis: Read through each sample solution. For each sample solution identify any mathematical errors made. Then answer the questions.

Sample Response: Alex
1. What assumptions has Alex made?
2. Are these assumptions correct? Explain your answer.
3. General comments regarding Alex’s method/solution.

Sample Response: Danny
1. Why do you think Danny starts at 0 small and 8 large boomerangs and stops at 6 small and 3 large boomerangs?
2. What piece of information has Danny forgotten to use?
3. Give any general comments you have regarding Danny’s method/solution.

Sample Response: Jeremiah
1. Is it correct to use equal signs in equations 1, 2, and 3? Explain your answer.
2. Why is Jeremiah’s solution incomplete?
3. Give any general comments you have regarding Jeremiah’s method/solution.
**Sample Response: Tanya**

1. What information does Tanya’s graph give us?

2. What do the slope and the y-intercept of each line mean in terms of the context of the situation?

3. Give any general comments you have regarding Tanya’s method/solution.

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C. Solving the system: Using a method seen in the sample work, or your own method, solve the system. How many large and small boomerangs should they make? How much money will they then make?

**Activity 4: Reflection**

1. While working on the Boomerangs task I could see that this problem could involve: (Mark the most appropriate activities).

<table>
<thead>
<tr>
<th>Calculating costs and times</th>
<th>Creating inequalities</th>
<th>Creating equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drawing graphs</td>
<td>Solving simultaneous equations</td>
<td>Controlling variables systematically</td>
</tr>
</tbody>
</table>

2.  

   My own method of solving was similar to 
   "______________'s method (insert name of sample response) because________________________
   _________________________________
   _________________________________
   _________________________________
   _________________________________
   _________________________________

   Or  
   My own method is different from **ALL** sample responses because ______________________
   _________________________________
   _________________________________
   _________________________________
   _________________________________
   _________________________________

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3. Of all the methods I have seen, the one that I prefer is: My own / Alex’s / Danny’s / Jeremiah’s / Tanya’s / Other (Circle) Because:
___________________________________________________________________________________________
___________________________________________________________________________________________
___________________________________________________________________________________________
___________________________________________________________________________________________
___________________________________________________________________________________________

4. What I found difficult about the mathematics in the problem and solving this problem was
___________________________________________________________________________________________
___________________________________________________________________________________________
___________________________________________________________________________________________

Cross Content Connection:
Economics: Supply and Demand - Companies use systems of equations when determining the best price to sell their products at in order to obtain the greatest profit. They understand that as price goes up, the number of people willing to buy that product will go down. However, the lower they price their item, the lower their profit. In addition, companies must also consider the demand of the seasons, locations, and community desire. With the weather getting warmer, what are some items that will have a greater demand? Does a change in demand always impact the price? What are some items that have been in high demand since Covid-19 has come to Illinois? How have their prices changed? What are some items that have not been in high demand since Covid-19 has come to Illinois? How have their prices changed?
Hints, terms and examples:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Words</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;</td>
<td>Greater than</td>
<td>( x + 3 &gt; 2 )</td>
</tr>
<tr>
<td>&lt;</td>
<td>Less than</td>
<td>( 7x &lt; 28 )</td>
</tr>
<tr>
<td>≥</td>
<td>Greater than or equal to; at least.</td>
<td>( 5 \geq x - 1 )</td>
</tr>
<tr>
<td>≤</td>
<td>Less than or equal to; no more than.</td>
<td>( 2y + 1 \leq 7 )</td>
</tr>
</tbody>
</table>

If you are stuck on Boomerangs:

A. Setting up the system:
   a. What do you know?
   b. What do you need to find out?
   c. What figures in the task are fixed?
   d. What is the greatest number of small/large boomerangs they can make?
   e. Have you used any unnecessary restrictions on the number of small and large boomerangs to be made?

B. Error Analysis
   a. What do you like about the work?
   b. What mistakes have been made?

C. Solving the System
   a. Can you organize the numbers of large and small boomerangs made in a systematic way?
   b. What would be realistic values to try? Why?
   c. Do you cover all possible combinations? If not, why not?
   d. How do you know for sure your answer is the best option?